Students' Reflections on Dispositions in a Mathematics Classroom

Teoh Sian Hoon[1], Parmjit Singh[1], Mazlini Adnan[2], Koo Ah Choo[3]

[1] Faculty of Education, Universiti Teknologi MARA, Malaysia
[2] Universiti Pendidikan Sultan Idris, Malaysia
[3] Multimedia University, Malaysia

teohsian@uitm.edu.my, parmj378@uitm.edu.my, mazlini@fsmt.upsi.edu.my, ackoo@mmu.edu.my
Tel: +60332584926

Abstract

This study investigates students' dispositions through a qualitative study that analyses students' reflective journal entries. It captures students' dispositions and describes how the reflective activities influence their engagement in mathematical problem-solving. The findings showed that the students considered the mathematical problems challenging, but their positive dispositions kept them engaged in learning. Engagement through effort and thinking algebraically with teachers' guidance was the crucial first step in problem-solving. Results from this study provide educators with a wealth of knowledge to develop learning dispositions that will encourage active thinking and engagement among students in mathematics classrooms.

Keywords: reflection; disposition; mathematics; engagement

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1.0 Introduction

Mathematics learning is an interwoven process that encourages students to think flexibly, but documentation of this process is necessary to enable teachers to detect students' learning difficulties and correct misconceptions. Using reflective writing will allow teachers to obtain more comprehensive individual feedback on the learning process and students' attitudes towards the learning. Getting students to produce journal entries promotes learning of Science, Technology, Engineering, and Mathematics (STEM). It increases students' engagement and connects their cognitive ability with the content (Thomas et al., 2020). This form of feedback also employs best teaching practices to improve students' disposition (Sansome, 2016). Hence, reflective journal writing enables teachers to obtain insights into the students' learning in mathematics classrooms.

2.0 Literature Review

Students engage in their learning differently, with attitudes that will psychologically affect their thinking and consequently their engagement (Mazana et al., 2019) and mathematics achievement (Casinillo et al., 2020). A multidimensional definition of attitudes towards mathematics has three perspectives: an emotional response to mathematics, a conception about mathematics, and a behavioural tendency concerning mathematics (Hart, 1989). Problem-solving and attitudes towards mathematics can be explained as a positive or negative psychological disposition towards mathematics (McLeod, 1994). Students' dispositions towards mathematics can be observed from their responses to situations, concepts, or objects based on the nature of mathematics. Fraser and Kahle (2007) emphasise that the learning environment significantly impacts attitudes to learning mathematics. The learning environment may have a considerable influence on students' responses to the content and concepts taught. It is argued that students use sense-making with flexibility in solving mathematics problems if they possess good dispositions towards mathematics, and hence they engage in learning (Graven, 2015). How they make sense, what students have experienced can be observed from their reflections (Lew & Schmidt, 2011). Previous studies (such as those conducted by Lew & Schmidt, 2011; Sperling et al., 2012; Kritt, 1993) examined reflection on how and what students learned. The findings revealed that reflection led to cognitive development, which directly affected academic performance. While insightful, these studies did not investigate the possible dispositions of students when engaged in problem-solving.

Engagement in learning mathematics needs attention as the process is complex, involving conceptual understanding, problem-solving, and integrations of various mathematical concepts. Within the processes, students' engagement and reflection are crucial to develop active learning. The engagement needs to be closely monitored. Limited reflection could indicate minimal involvement. Engagement in the process of learning involves significant thinking. High engagement with more thinking which shows healthy dispositions towards mathematics, is ultimately preferable in learning concepts. Reflection performed within the learning processes would create awareness of learning processes, progress, and active engagement (Di Stefano et al., 2016). It provides insight into complex learning, evident from a willingness to reflect on the thinking and develop constructive dispositions towards mathematics (Costa, & Kallick 2008). However, this pedagogy needs to be carefully planned to ensure that the learning tasks are appropriately structured to encourage students to develop even more thinking through engagement in learning.

The attention devoted to students' difficulties in performing various levels of tasks in mathematical learning has motivated researchers to conduct studies on students' engagement. Interest has turned to students' achievement in solving high-order thinking questions, defined as word problems in mathematics. It is observed that students' low achievement in mathematics tends to be related to their failure in understanding mathematical concepts and schools' difficulties in providing the students with prerequisite knowledge. Even more challenging mathematical problems are presented at the next stage of learning. Hence, it is unsurprising that students are encountering difficulties following more advanced lessons. Their teachers are aware of their difficulties and have done much to help. For example, Teoh et al. (2010) has created a motivational instrument to gauge learning. Dewey (1933) emphasises the need to take responsibility and play significant roles in an individual's performance to ensure that students immerse themselves fully in the learning activities in a mathematics classroom. Certainly, structured activities on comprehensive learning contents create a nurturing environment for students to take responsibility for monitoring themselves in solving challenging mathematics problems (Teoh et al., 2017). Teachers, therefore, need to provide as many opportunities as possible for students to actively master their learning content, primarily through doing, talking, reflecting, discussing, observing, investigating, listening, and reasoning (Coopley, 2000). Such activities enable students to work meaningfully on learning tasks. In a supportive environment, the teacher activates students' prior knowledge through questioning to develop transferable thinking skills. The experience of being in such a supportive learning environment may arouse students' interest and encourage them to think about what works in solving mathematical problems (Davadas & Lay, 2020). The thinking process directly increases students' engagement.

Many suggestions have been put forward for designing meaningful activities to increase the time students spend on a task and engage them in learning. Lumsden (1994) asserts that students are more engaged when given significant and worthy problems or course materials related to real-life situations. Similarly, Parmjit and Teoh (2010) recommend ensuring content adequacy of school learning materials, namely the textbooks, used in classroom activities. Contents need to be carefully designed so that students are acquainted with real-world word problems. Research has shown that students enjoy learning when they are given control over learning, for example, when they can employ strategies to solve problems (Brooks et al., 1998). Tasks given to students must have ample opportunities to apply knowledge gained, provide a sense of competency, and help students build knowledge. Students also need to contemplate their experiences to consistently recall good practices and important points, as highlighted by Anderman and Midgley (1998) and Coopley (2000).

Reflection on classroom learning activities keeps students engaged and increases their time on-task. The focus of time-on-task indicates persistence in working through the challenges of solving any mathematical problems. High engagement implies the use of strategies. Moreover, efforts made in written communication will further encourage active thinking when it involves discussions that require students to make their best attempt while being engaged in their learning and display an understanding of the thinking processes involved. Written reflections are beneficial at various levels of the teaching and learning process (Johns, 2000). A written reflection alters the monitoring of individual experiences, leading to a new understanding of the process of learning (Boud et al., 1985). Also, reflection becomes a response to the learner's learning experiences, facilitating the formation of the habit of linking and constructing meaning from experiences (Feuerstein et al., 1980). When students are engaged in writing their reflections on activities in a mathematics classroom, they communicate within the text. The written communication describes their learning activities in the classroom and usually involves the recollection of their learning processes, requiring them to think beyond what has happened. Therefore, reflection is a technique for aiding and reinforcing learning. Active learners who put effort into reflective writing are always aware of their learning, enabling them to control their learning. When they are actively involved in the reflection, the persistence in the engagement is intrinsically motivating and reduces the likelihood of dropping out from school or experiencing boredom in classrooms, thus developing a positive disposition among students (Yazzie-Mintz, 2010; Dunleavy & Milton, 2010).

Learning disposition allows students to focus on improving learning processes and reflect on how they carry out learning. It helps them to navigate a good learning pathway. The reflections include their sense of themselves in learning, curiosity, making connections, intuition, being aware of their understanding and resilience (Deakin, 2012; Deakin & Yu, 2008; Deakin et al., 2004).

In some classrooms, reflective writing is used as an assessment to provide information about students' advancement in learning. The reflective writing offers in-depth observation of students' progress in the learning process. The opportunity to follow through the learning process helps students monitor their learning and become active respondents. In other words, the students establish their learning and practise applying reflective journal writing (Ormrod,1999). Hence, the use of reflective writing has always received the support of educators. Peltier et al. (2005) and Williams (2008) investigated the implementation of reflective journal writing, and they found that it is a useful way to assess the processes of learning and provide input for university course development.

Furthermore, students' willingness to engage reinforces teachers' desire to foster students' cognitive development and acquisition. Hence, it is essential to discover how students' behaviour or disposition underpin mathematics learning directly from students' reflections. Reflective writing provides information on the created learning environment. The ideal environment to develop mathematical disposition focuses on students' tendency to act and engage positively by showing their interest, willingness, and perseverance in solving challenging mathematical tasks. Much research has examined the relationship between students' mathematics results and their engagement. However, it is still not clearly understood the role students' disposition plays in solving a mathematical problem. This study examined students' reflections in terms of their engagement in solving mathematical problems. It specifically analysed students' dispositions towards mathematical problems. The result may provide a guide to foster favourable dispositions towards mathematics among students for more engagement and active learning.

3.0 Methodology

A specific classroom setting has been designed to conduct the activity-based mathematics lessons to achieve the objectives of the present study. The contents of the mathematics lessons required the use of mathematical thinking skills to solve mathematical problems. The mathematics lessons were part of a compulsory course for the students. The students received guidance on engagement in activities providing opportunities to solve word problems in the mathematics classroom. This study focused on a small group of the population which comprised first-semester science-based Malaysian diploma students at a public university. These students had registered for a mathematics problem-solving course at the university. This study involved a total of 120 students who were randomly selected from the population. The main instrument in this study was reflective writing. The participants wrote their reflections after the completion of the activities. Mathematical problems used for the activities were adopted and adapted from Parmjit's (2009) instrument. The following mathematical problem is among those assigned to the students:

If it takes six men to paint a house in 21 days, how many men will be needed to paint the house in 14 days?

Problem-solving skills were taught to the participants using the heuristic approach to solving mathematics problems relating to algebra, geometry, and statistics in each of the lessons. Then, problems or activities to be solved were given during and after each lesson. After five weeks, they reflected on the classroom activities or discussions in the course. The study employed a qualitative approach to collecting data by requiring each student to keep a reflective journal. The students were required to reflect on the activities done in the classrooms for the five weeks. The students submitted their reflective journals immediately after completing the activities or lessons in the classroom through an online feedback system. The students' feedback in the reflective journal was analysed based on the contents stated in Table 1. All the responses were carefully read, and codes were assigned thematically. Students' responses to each item were presented to illuminate the findings. Results that highlighted engagement were further interpreted in terms of the actions taken and dispositions displayed in the classroom. For example, the journal entries were given systematic labels, for example, R1 for the first respondent, R2 for the second respondent. The instrument, adapted from Cavilla's (2017) study, is presented in Table 1.

Table 1. Questions for the reflection (adapted from Cavilla's (2017) study)

Indicator	Items	Questions for Reflective Writing in this study
Academic Performance	How has this activity changed the way I think about this concept? Does the result of this assignment live up to my expectations?	What are the common strategies you use in seeking solution to the given problems? What challenges did you face in this course? Please explain.
Academic Motivation	Did I truly do everything I could to succeed in this task? If I were to revisit this assignment in the future, what would I do differently?	Are problems similar to the ones in your previous school examination? Why? Were the problems easy? Did you know how to solve the problems as you glanced through them? Please explain
Affective Motivation	Do I feel that my work on this assignment has the potential to influence others? How has this assignment fueled my thirst to learn more about this topic?	Did the problems motivate you to search for a solution? Why?

4.0 Results

The students' reflections helped to answer the following research questions:-

Research question 1: What are the students' dispositions towards mathematical problems?
Research question 2: How do the students engage in solving mathematical problems?

Data that will answer the first research question were collected from each student's reflections labelled 1 – 3. Data that will answer the second research question were collected from reflections labelled 4 -5. Table 2 presents a summary of the questions to guide the five reflections.

Table 2. Question for the reflection

Reflection	Research Question
1. Are problems similar to the ones in your previous school examination? Why? Were the problems easy?	*Research question 1:* What are the students' dispositions towards mathematical problems?
2. Did you know how to solve the problems as you glanced through them? Please explain.	
3. Did the problems motivate you to search for a solution? Why?	
4. What are the common strategies you use in seeking solution to the given problems?	*Research question 2:* How do the students engage in solving mathematical problems?
5.What challenges did you face in this course? Please explain	

4.1 Findings for research question one (reflection one)

The findings were obtained from the analysis of the students' responses to these prompts to guide the reflection:-

Reflection one: Are the problems similar to the ones in your previous school examinations? Why? Were the problems easy?

Using Reflection One, the study investigated students' perceptions of the course content and students' learning strategies. Indirectly, their disposition towards mathematics problems was investigated as well. A few codes were assigned based on the students' responses. One was 'Yes', indicating that the students agreed that they had similar experiences in answering the questions and responding to the activities when they were in secondary school, while 'No' indicated the opposite, and 'Sometimes' indicated that their experiences were minimal. Table 3 shows that 14% did not respond, and only 86% responded to the three questions. Hence, the first reflection results were reported based on 86% of the respondents' reflections.

Table 3. Feedback on the 1st reflection

Categories of responses	Number of students	Percentage of students
Yes	34	28
No	41	35
Sometimes	28	23
No Response	17	14 (86% of the respondents replied)
TOTAL	120	100

From the reflections, 35% of the students reported having no experience solving similar mathematical problems in the classroom. The responses revealed that the questions given were different from the questions they had encountered in their secondary school years. Also, slightly more than half of the students (28% and 23%) reported that they had answered similar types of questions during their secondary school years. The results showed that most of the students agreed that the questions or problems were not new. Nevertheless, they needed to apply some skills based on their understanding of related mathematics concepts. The reflections of more than half of the students (51%) showed that students were aware that mathematics concepts were interrelated. Even though some problems were new to the students, they revealed that the questions were closely related to the concepts and content taught in the secondary school mathematics curriculum. However, students still require exposure to a diverse range of problems to help them master specific concepts. The teaching and learning of mathematics should always be related to problem-solving; therefore, students must solve as many problems as possible to master specific mathematical concepts or strengthen their understanding of the various concepts in mathematics. It is argued that the teaching of mathematical concepts and skills needs to include many different tasks to present different challenging, structured problems to students to facilitate learning (Elbers, 2003). When tackling these problems, students will draw on their mathematical knowledge to find solutions from various aspects. In this study, the mathematics test in this study required respondents to apply basic mathematical knowledge. Even though the word problems were new to the students, they would be able to solve them. If they could use the concepts learned earlier, they would be able to work out the solutions. Through observation, the familiarity with mathematics questions is also subjectively defined. All mathematical problems are related to previous learning in primary school and lower secondary school during which algebraic thinking was taught. A respondent explained:-

R: The questions are not similar to the questions I have done before. They look like KBAT [high order thinking] questions. One must (be) good in basic knowledge, especially algebra, to find that not so difficult to solve (sic).

The results showed that most students acknowledged that the questions or problems were not new to them. They just needed to apply some problem-solving skills based on related concepts. Nevertheless, some students confessed that they were not familiar with the questions. The nature of problem-solving that uses a non-routine structure had caused some of the students not to be familiar with the questions. Nevertheless, it was acknowledged that algebraic knowledge was essential in solving the questions. Therefore, students need to be aware that problem-solving requires them to think beyond "isolated knowledge" and connect concepts and adopt flexibility in thinking. The analysis revealed that most students had the opportunity to solve complex or higher-order thinking problems in school. They acknowledged that building up algebraic knowledge was crucial for them to solve these problems. This finding indicates that students need to be equipped with transferable skills to have the flexibility in tackling word problems, facilitated by applying knowledge of algebra. Fundamental principles of algebra are introduced in primary school and developed in the secondary school years. Hence, students' flexibility in using algebraic knowledge depends greatly on their primary and secondary school learning experiences. The students' flexibility underpins their sense-making in numerical knowledge (Graven, 2015). The valuable reflections increase their awareness of their cognitive development in sense-making within the algebraic knowledge (Lew & Schmidt, 2011) and enhance positive dispositions in the flexibility of recalling the knowledge (McLeod, 1994). The first finding confirms that mastery of foundational concepts of algebra is crucial in primary and secondary schools as it develops sense-making to promote deeper thinking and engagement, which fosters a positive disposition in their attempt to work out mathematical problems.

4.2 Findings for research question one (reflection two)

The following results were obtained by analysing the reflections of students in response to this question:-

Did you know how to solve the problems as you glanced through them? Please explain.

Table 4 shows that 13% of the respondents did not offer their thoughts on the matter. Only 87% of the respondents responded to the question and provided explanations. Hence, the results for the first reflection were based on 87% of respondents.

Table 4. Feedback on the 2nd reflection

Categories of responses	Number of students	Percentage of students
Difficult	18	15
Engage thinking	36	30
Good experience	2	2
Guidance	27	23
Need more exposure	1	1
Time spent	20	16
No response	16	13
TOTAL	120	100

Table 4 shows the range of responses produced by the students. The response suggests that students need to think more deeply to solve the problems. The difficulty they experienced was apparent in the reflections. For instance, the students explained that they needed some guidance and time to answer the questions. A respondent (from R7) wrote:-

"For the second week, I think it's getting a little bit easier for me to solve the problems. But I still need some more practice and guidance to solve the problems given. It's because I can't solve it just by a single glance and need some extreme brainstorming to find the solution. I hope I will get better from time to time and get to use it as fast as I can" (sic).

The findings indicate that the students need much more support than given, possibly from teachers and more able friends and classmates. Indeed, cooperative learning is encouraged in the mathematics classroom, as highlighted by Johnson and Johnson (1975) and Davidson (1990). Students are more likely to receive help in different forms when learning with their peers. Students who provide reasons when answering questions are considered active thinkers. The reasoning will help others learn (Coopley, 2000), although they still need guidance from their teachers (Ma, 1999; Stigler, 2004; Weiss et al., 2004). About specific strategies used, R8 explained that he always considered an alternative to solving problems, as taught by teachers in his secondary school.

R8: I need confirmation from a friend, or any teacher about what I did was right. But, when there is nobody to help, I think on my own by thinking about what I have done before. I think mathematics is something that we need to think about. Teachers always reminded us that think before asking (sic).

This reflection shows that students need guidance in solving mathematical problems and developing their thinking. With guidance, they can apply some familiar strategies related to their learning. The individual's ability to think deeply based on available knowledge is an essential characteristic of active learning – described as flexible or generative, according to Carpenter et al.(1996). Individuals with generative ability to learn are capable of transferring skills and knowledge to solve other problems. Hence, teachers' guidance will help students engage actively in learning and develop a positive disposition to think deeply. The second finding draws attention to the facilitative effects of guidance on student engagement.

4.3 Finding for research question one (reflection three)

The reflections based on these questions yielded some thought-provoking insights:-

Reflection Three: The students reflected on the following questions: Did the problems motivate you to search for a solution? Why?

When they were asked whether the questions motivated them to search for the solutions, Table 5 shows that 65% of respondents' implied engagement, such as "effort"(34%) ", interested" (16%) "think" (15%), suggesting that the problems encouraged them to find solutions. It was enlightening to learn that they found that the questions interesting. The students were 'interested' (16 % of the respondents) to answer the questions, and that the questions motivated them to 'think' (15% of the respondents). Very few (2%) of the respondents found that they were not motivated to solve the problems since they were too difficult.

Table 5. Feedback on the 3rd reflection

Categories of response	Number of students	Percentage of students
Difficult	2	2
Effort	41	34
Interested	20	16
Think	18	15
No response	39	33
TOTAL	120	100

Generally, the questions prompted them to expend effort to solve mathematical problems. The students' efforts to solve the problems encouraged them to think more deeply to understand the problems (Halpern, 1998). The development of thinking is evidenced by the number of questions asked to seek clarification and the rapport between teachers and students. Deep thinking is promoted through interaction (Coopley, 2000; Reinhart, 2000). Cognitively, they took responsibility for their learning by monitoring themselves and expending effort (Coopley, 2000; Zimmerman & Schunk, 1989). The analysis revealed that students practised self-monitoring and were active learners, engaged in applying mathematical concepts to solve problems.

4.4 Finding for research question two (reflection four)

Reflection four provides insights into the strategies that students had employed in response to the following question:-

Reflection Four: What are the common strategies you use in seeking solutions to the given problems?

In their reflections, students recalled the strategies used to solve the questions. According to Schoenfeld (1985), employing appropriate strategies can help students to develop their thinking skills. Table 6 shows the strategies that students reported using. 27% of the respondents reflected that the first step in solving the problem is understanding the given task. This effort showed that the students spent time carefully reading the questions and understanding a problem as much as possible. The strategies in achieving comprehension included reading a question several times, interpreting, contextualising, or comparing solutions to previous questions. In this study, many respondents (15 %) reported that they drew on prior knowledge to solve a problem. Another strategy employed by 14% of the students was to examine the patterns in solving problems. Other than that, other strategies included using taught formulae (9%), visualisation (7%), and discussion with peers (8%). Students used strategies selectively. Only very few students (6%) applied the complete gamut of strategies in solving problems - understanding the problem, planning a strategy, implementing the strategy, and rechecking, following the process described in Polya's (2004) teaching and learning of problem-solving.

Table 6. Feedback on the 4th reflection

Categories of response	Number of students	Percentage of students
Discuss	9	8
Experience	18	15
Formula	11	9
Pattern	17	14
Process	7	6
Understanding	33	27
Visual	8	7
No response	17	14
TOTAL	120	100

4.5 Finding for research question two (reflection five)

The following results were derived from the analysis of the reflections, which were prompted by the following question:-

Reflection Five: What challenges did you face in this course? Please explain

The types of challenges that the students had reportedly experienced are displayed in Table 6. More than two-thirds of the respondents (71%) found the questions challenging. Very few students (6%) found them interesting, and some (3%) reported resorting to collaboration to find solutions to the questions even after five weeks of lessons. It was clear that the majority of the students found the tasks given were too difficult. However, despite the challenges, students reported expending their best efforts to understand and solve the problems.

Table 7. Feedback on the 5th reflection

Categories of response	Number of students	Percentage of students
Collaboration	4	3
Difficult	85	71
Interesting	7	6
No response	24	20
TOTAL	120	100

5.0 Discussion

Reflections One, Two, and Three elicited students' feedback to uncover their dispositions towards mathematics. The findings derived from the in-depth analysis of students' reflections have shed light on students' attitudes and ways of solving challenging word problems. These findings provide teachers with valuable input for engaging the students in and outside mathematics classrooms. The findings show that students were aware that the math concepts are interrelated. However, teachers need to highlight the links between concepts taught to heighten the awareness. Even though some problems appeared new to the students, the questions assessed students' understanding of the secondary school syllabi concepts. Besides, foundational algebraic knowledge was found to be crucial for students in solving mathematical problems. The flexibility in thinking using related fundamental algebraic knowledge was considered facilitative in providing structured knowledge for solving mathematical word problems. The findings obtained by the present study are compatible with those of previous research, which drew attention to the importance of flexibility in mathematical thinking. Students who practise flexibility in solving problems will always attempt to make sense of problems and find solutions. Masini and Taylor (2000) assert that a student's mathematics achievement depends on conceptual understanding and the flexibility to relate the foundational mathematics concepts. When students can understand questions flexibly, they demonstrate familiarity with the mathematical concepts. Research has also shown that students' familiarity with mathematical concepts motivates and engages them in further learning. Prasad (2014) investigated the concept of familiarity and flexibility in terms of number sense. It was found that by thinking mathematically, students could manipulate solutions conceptually. They applied requisite knowledge to enable them to think structurally.

The findings obtained from the present study have also further illuminated research conducted on students' disposition towards mathematics. Mathematical thinking rests on students' disposition towards mathematics (Thomas et al., 2020). Students who appreciate the characteristics of mathematical communication, namely the terms, concepts, and symbols defining them, will be favourably disposed towards any mathematics questions. The ease of applying concepts shows they have a strong disposition to communication within the text (Teoh et al., 2017). They also apply fundamental mathematical knowledge in manipulations of physical experiences. The manipulation process involves a conceptual mechanism to understand the situation and language in various areas (Lakoff & Numez, 2000). Although students found the tasks challenging in this study, they also revealed a disposition to think algebraically to persist in engaging in the mathematics classroom (refer to Figure 1). This favourable disposition was also conveyed through the need for extra time in understanding and communicating within the text. These revelations suggest that they were engaging in high-level thinking. Studies have also shown that high-level tasks are often complex, and more time is required for the problem solvers to think (Elbers, 2003; Lumsden, 1994). In this study, students (as reported in Reflection Two) who lacked the disposition towards solving mathematics problems reportedly required more guidance. Therefore, it is important that teachers carefully monitor and support their students to reassure them that the concepts were familiar and different. The guidance will boost students' confidence to be more favourably disposed towards solving mathematical problems.

The findings from this study have also validated previous findings on the importance of constructing mathematical tasks to engage students' thinking level more deeply of the problems presented to them. This study revealed that students engaged in thinking (30%) and were willing to spend time dealing with the problems (16%). These findings showed that the mathematical tasks were appropriately constructed as nearly half of the students in the study expended effort in tackling the problems. The importance of constructing mathematical tasks that enable the development of thinking and reasoning flexibly is also highlighted by Heibert and Wearne (1993) and Stein et al. (2000). Even though the students reportedly faced difficulties understanding the questions, many were still motivated to find solutions. Thus, by constructing meaningful tasks, educators can instil dispositions and develop more in-depth thinking in the learning process of mathematics. Together with the teacher's guidance and other forms of support (for example, peer teaching), students' dispositions can be strengthened (refer to Figure 1).

The findings imply that each mathematics teacher needs to be informed by the student's background, abilities and disposition. Teachers' knowledge of students will help them design mathematics lessons that will fully engage them (Peltier et al., 2005; Williams, 2008). Teachers also must ensure that these lessons carry meaningful and effective content to develop and optimise students' dispositions towards problem-solving (Kusmaryono et al., 2019). Thus, skills in teaching mathematics must include knowledge of best classroom practices and excellent content knowledge. In this study, although the contents of lessons were perceived to be difficult, students remained motivated and persisted in solving the problems. These findings draw attention to the role of intrinsic motivation of wanting to succeed at problem-solving and extrinsic motivation from teachers and collaborative learning, as presented in Figure 1. Despite the challenges, the students expended effort in thinking. The positive dispositions towards solving mathematical problems can be seen in the effort made, time spent, and the need for assistance, as shown in Reflection Three. It is argued that students will show solid dispositions through teachers' support. Students revealed that they had put much effort into solving challenging but interesting problems. To perceive the questions as interesting would imply that students possess a positive disposition towards them. They reacted and thought positively in problem-solving, as seen from the considerable effort made and their thought processes. All these are indicative of positive attitudes to learning. As shown in their dispositions, the positive ways of thinking had enabled them to engage in the learning.

6.0 Conclusion

These findings have highlighted important positive attributes needed to tackle problems that require higher-order thinking. Students may perceive that solving mathematical problems is difficult, but their dispositions - shown through self-directed engagement and flexibility of learning are key to mastering mathematical concepts. However, teachers need to recognise that this engagement needs to be teacher-led for some students. Engagement, fuelled by students' effort and teachers' guidance, is crucial in solving problems. Thus, educators must instil dispositions that encourage deep mental processing in the learning of mathematics. The figure below sums up the major findings of this study.

Engagement

- shaped by experience
- guidance
- time-spent

} Disposition (the mathematics tasks are difficult, but think algebraically with effort)

Figure 1 Summary of the findings

This study, albeit small-scale in nature, has found that students had experience solving similar types of questions, but they still found problem-solving in mathematics particularly challenging. Nevertheless, they made a substantial effort, as seen from the strategies reported, time spent, and the mental processing to solve the problems. Finally, this study has also confirmed that cognitive development is shaped by learning experiences. Therefore, teachers could make learning experiences meaningful and engaging in developing students' mathematical thinking skills by constructing meaningful tasks for problem-solving, developing a good disposition to solving mathematical problems, monitoring students closely, and offering assistance whenever possible. Learning mathematics is a complex activity as it involves complex strands. Still, it is possible to equip individuals with mathematical skills by building a positive disposition towards self-learning and monitoring.

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Article Contribution to Related Field of Study

This study provides insight into ways of developing positive dispositions in the teaching and learning of mathematics.

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