How Do 13-Year-Olds in Malaysia Compare Proper Fractions?

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Abstract

In an effort to understand the problems and difficulties students faced in learning fractions, a study was carried out on Form 1 students from four different types of secondary schools in Perlis, Malaysia. Five different questions on proper fractions were constructed involving two fractions with equal and different denominators, and more than two fractions with different denominators. A Rasch analysis was done on the responses to help assess their level of conceptual understanding.The whole number concept was very dominant as some of the students treated the numerators and denominators separately, showing their confusion and unstable thinking in fractions learning.

Keywords: Fraction understanding, proper fraction, whole number, Rasch analysis

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1.0 Introduction

Fraction is a number that expresses part of a group. Primary students in Malaysia are exposed to the fractions learning for four years beginning as early as Year 3. In Year 4, students are introduced to proper fractions and are required to name and write proper fractions with denominators up to ten, express equivalent fractions to proper fractions, add two proper fractions with denominators up to ten, and subtract proper fractions with denominators up to ten (Mathematics Year 4, 2006). By the end of Year 6, these students should be able to understand proper fractions as well as improper fractions and mixed numbers, and be competent enough with the algorithms that are connected to fractions (Mathematics Year 6, 2006).

Conceptual understanding of fractions is a prerequisite to dealing with other areas of mathematics. The conceptual understanding develops when students are able to connect between concepts and procedures and can explain why some facts are consequences of others (National Research Council, 2001; Wong and Evans, 2007). Thus, misconception in fraction learning may cause problems with other domains in mathematics such as algebra, measurement, ratio and proportion concepts (Behr, Lesh, Post, & Silver, 1983). In an early study by Pearn and Stephens (2002, 2006), students were found to demonstrate inappropriate use of whole number thinking strategies to fraction problems. Students who were able to order whole numbers failed to realize that this thinking strategy was only applicable for fractions with same denominators. Since fractions make up an integral part of the Malaysia primary school curriculum, it is therefore important to examine whether by the end of Year 6, students do have a good conceptual understanding of fractions. This paper would specifically present how Form One students compare proper fractions after completing the four years of learning fractions at primary school level. Are students in Malaysia able to show understanding in comparing proper fractions?

2.0 Literature Review

Fractions is a topic many students find difficult to learn (Behr et al., 1983). This is compounded by the fact that it is also a topic many teachers find difficult to understand and teach (Post, Cramer, Behr, Lesh, and Harel, 1993; Clarke, Roche and Mitchell, 2007; Tengku Zainal, Mustapha and Habib, 2009). It is foreseen therefore that learning fractions presents considerable challenges and difficulties for students throughout their school years which would later cause them to have problems with other domains in mathematics (Behr et al., 1983). An analysis of students' performances in major government examination would shed some light on students' conceptual understanding of mathematics. Students' performance in the Ujian Pencapaian SekolahRendah (UPSR) in 1996 was below the satisfactory level especially on fraction questions (Lembaga Peperiksaan, 1997; Tahir, 2007). Later analysis showed that students did not understand the basic concepts of fractions in the UPSR for 2001 (Lembaga Peperiksaan, 2002; Tahir, 2007). At the secondary level, in the 1994 Penilaian Menengah Rendah (PMR), only 565 of students were able to correctly answer 2/3 + 1/6. The common mistake made by weak students was 2/3 +1/6 = 3/9 (Lembaga Peperiksaan, 1995; Tahir, 2007). The Lembaga Peperiksaan (1996) also reported that in 1995, only 57% of PMR candidates managed to answer correctly 2/3 + 3/5 despite among mistakes 2/3 + 3/5 = 2/5. In the same paper, only 50% of the students were able to answer correctly questions involving fractions, decimal numbers and whole numbers, and there were students who converted 1 5/8 to 15/8 (Tahir, 2007).

State and national assessments suggested that students have often failed to produce a workable concept of fraction (Behr, Wachsmuth, Post and Lesh, 1984). That might be a good reason why many western education systems now explicitly discourage teachers from introducing algorithms before the students develop in-depth conceptual understanding of part-whole relationships within the number system (National Council of Teachers of Mathematics, 2000; Young-Loveridge, Taylor, Hàwera and Sharma, 2007). Regrettably, study indicated that the teaching activities in Malaysian classes start and end with a collection of symbols and abstract mathematical terms with so many rules and tips to memorize. It was no surprise then when teachers were able to point out mistakes such as 2/6 + 1/6 = 2+1/ 6+6 or 2/3 – 1/6 = 2-1/6-3 = 1/3 (Tengku Zainal et al., 2009).

A systematic examination of the kinds of errors in fractions that students made conducted by Brown and Quinn (2006) detected that out of 143 students, 27 students were unable to find a common denominator when adding unlike fractions, more than two-thirds of them commit “add across” errors by adding the numerators followed by adding denominators, a fifth of them showed misconceptions related to equivalent fractions and only half of them managed to add 5/12 and 3/8 successfully (Young-Loveridge et al., 2007).

According to Behr et al. (1984), students will be able to compare the relative size of a pair of fractions A and B, whether A is equal to B, A is less than B or A is greater than B, unless they have insecure rational number concepts which tend to have a continuing interference from their knowledge of whole numbers. In his study, a child said “*the size of the piece is the same, the number of pieces is important”* in explaining why he gave 3/7, 8/7, 10/7 answer of ordering fractions from smallest to largest. In the same study, a response by a Grade 4 student when asked why two fifths was less than two thirds was interesting to observe*: “There are two pieces in each, but the pieces in two fifths are smaller. So a smaller amount of the unit is covered for two fifths*”.

In another example however, the student demonstrated whole number dominance by saying that one third was less than one fourth because three was less than four. Another interesting explanation which may also lead to the fake answer should also be noticed. From probing interview done in earlier studies on fourth graders for instance, the child responded 9/13 was less than 4/13 when comparing these two fractions because *“four pieces are so big, nine pieces have to be smaller to fit the whole”* (Behr et al., 1984).

3.0 Methodology

This study involves 288 students from five randomly selected secondary schools in Perlis, Malaysia, representing four different types of schools, which are cluster school, residential school, religious school and two ordinary schools. Two sets of instruments were developed, namely the Assessment of Fraction Understanding (AFU) and the Probing Fraction Interview (PFI). The AFU which was adapted from an earlier AFU instrument built by Wong, Evans and Anderson (2006) and was constructed based on the UPSR format and the requirements of the Primary School Curriculum Specifications. In about one and a half hours, students need to answer 33 subjective questions in the AFU that contained 40 items on Fractions covering three major components: Proper Fractions (66.65%), Improper Fractions (6.67%) and Mixed Numbers (26.67%). Under Proper Fractions component, students were tested on their ability in naming and writing fractions, expressing and writing equivalent fractions for proper fractions, comparing proper fractions, and adding and subtracting two proper fractions.

A Rasch analysis using WINSTEPS software was done on the AFU to determine the level of conceptual understanding among the students and testing the responses reliability. The PFI used five questions selected from the AFU, containing Kieren's (sorry this is correct) five sub-constructs of fraction interpretation: Part-whole, Measure, Operator, Ratio and Quotient. Five students from each school were randomly selected for the interview session which was video-taped and transcribed. The PFI was used for further identification of common errors and problems with item interpretation, and to understand the thinking and the procedural processes plus the algorithms undertaken by the students in answering the five selected questions (Wong et al., 2006; Abdol Razak, Noordin, Dollah and Alias, 2010).

4. 0 Results and Discussions

Students were given five different questions with the objective to compare proper fractions with the same or different denominators. Rasch analysis done on the responses showed that all the questions lay below the mean and were getting closer and closer to the mean, meaning that most students were able to succeed and rarely missed them. Missing them once in a while however was not impossible. The discussion presented would be divided into three main sections to see the students' ability in comparing fractions with same denominators, ability in comparing two fractions with different denominators, and ability in comparing more than two fractions with different denominators. In terms of hierarchy, the discussion started with the easiest question and moved to the toughest questions as indicated by the analysis on the item map.

4.1 Fractions with Same Denominators

In comparing fractions with same denominators, students were asked to choose the fraction with the biggest value from a set of 3/9, 7/9, and 5/9. From our study we had 231 (80.21%) students giving 7/9 as the answer. Having "the same size of the 9 piece" which is nine, they were able to recognize "the largest number of pieces" that is seven, to get the fraction with the biggest value. This finding agreed with "the importance of the number of pieces when the size of the piece is the same", as most of the students were able to manipulate fractions with the same denominator in this manner.

There were however 50 students who answered 3/9 There were however 50 students who answered , the opposite (meaning smallest) of the correct answer, four students answered 5/9 and the others 9 (1.04%) gave incorrect and inconsistent answers. Was there any chance of misinterpreting the word "biggest" as "smallest" hence misleading those students to the wrong answer? If this was the case, then it should explain on why there were 17.36% of students considered 3/9 as the biggest fraction, but that would certainly not explain why 1.39% of the students answered 5/9.

4.2 Two Fractions with Different Denominators

Seeing quite a high percentage of success in question involving fractions with the same denominator, do the students have a good conceptual understanding in comparing fractions with different denominators? Let's move on to the next easy question as follows:

Which fraction has a bigger value, 3/10 or 3/5?

Almost 76.04% of the students were able to correctly answer 3/5.

In this question, there were some students who were unable to treat the numerator and denominator as being connected to one another. Perhaps this was why 64 (22.22%) of them gave the response 3/10. Since the numerators for 10 both of the fractions were equal, these students applied the whole number concept by focusing only to the denominators, thus choosing the false answer.

This conceptual misunderstanding happened again in the next question when students had to decide on which of the two fractions was smaller between 1/5 and 1/7. This question also required students to compare two proper fractions.

Interestingly the percentage of incorrect answer for this task increased to 32.99%. Thus, what what was causing confusion to these students? Notice that the first question was easier since it involved denominators of five and ten, which were multiples of each other. The second question on the other hand involved different denominators, five and seven, and students would need to bring these fractions to a common denominator before they were able to compare the two fractions. Was the process disturbing the students' unstable understanding, thus resulting in a higher percentage of incorrect answer?

4.3 More than Two Fractions with Different Denominators

Preceding finding showed that there were some students who were not quite able to compare two fractions with the same numerator but different denominators. These students were expected to have more problems in handling the next two questions since the given set of fractions was bigger. In a set of 1/2, 1/6, 1/9, 1/5, 1/7 for instance; only 183 (63.54%) students were able to arrange the set given in ascending order. The rest of the students, with the exception of two students who did not indicate any response to the question, ordered the fractions in the following manner, namely ½, 1/5, 1/6, 1/7, 1/9 implying that students were focusing on the denominators. The whole number concept was very dominant in play here, the larger the denominator, the bigger the fraction. This agreed with results of probing interviews handled by Pearn and Stephens (2007) for the case of larger-is-bigger-thinking.

Having a set of 1/3, 1/5, 1/8, 1/7; students again showed a problem in picking out the fraction with the smallest value. There were 176 (61.11%) of them able to answer correctly. As in the prior question, it appeared as if larger-is-bigger-thinking was in play again here. This would answer why 71 (24.65%) students answered 1/3 as the smallest.

5.0 Conclusion

From the findings presented earlier, the percentage of correct answer dropped from 80.21% for the easiest question to 61.11% for the toughest question (see Table 1). Based on our own rubric, if the percentage of success is not less than 70%, then the students are considered as having a good conceptual understanding of comparing proper fractions.

Table 1: Percentage of correct answer for different objectives of question

Objective	Question	Percentage of correct answer
To compare proper fractions with same denominators.	Which of the following fractions has the biggest value? 3/9, 7/9, 5/9	80.21
To compare two proper fractions with denominators of multiple of each other.	Which fraction has a bigger value, 3/10 or 3/5?	76.04
To compare two proper fractions with denominators not of multiple of each other.	Which fraction has a smaller value, 1/5 or 1/7?	67.01
To compare more than twofractions with different denominators.	Arrange the following fractions in ascending order: ½, 1/6, 1/9, 1/5, 1/7	63.54
To compare more than two fractions with different denominators.	Determine the fraction with the smallest value from the following: 1/3, 1/5, 1/8, 1/7	61.11

The students have a good conceptual understanding for the first two questions as depicted in Table 1. In a given set of three fractions with equal denominator, most of the students were able to compare these fractions and decide on the largest value of fraction from the set. With a drop of 4.17% of success, they could still compare two fractions with different denominators. However, these students started to show instability of (adjective needed hereof?) understanding in comparing fractions with different denominators as they could compare fractions with denominators that are multiple of each other better than fractions that needed to be converted to a common denominator. Seeing this, we expect that the students could have much more trouble in handling a set of more than two fractions with different denominators. Our expectation was proven true when the percentage of success decreased further to only 61% to 64. It appeared as if larger-is-bigger-thinking was in play, showing not a very good conceptual understanding of comparing proper fractions among 13-year old students in Malaysia.

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